

# Dense European velocity Field and ETRS89 positions and velocities of the EPN stations

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## Abstract

The EUREF Technical Working Group has initiated a project to establish a Dense European Velocity Field (DEVF) whose main objective is to ensure the long term maintenance of the European Terrestrial Reference Frame 1989 (ETRS89). As contribution to this DEVF project, we analyse the EPN weekly combined solutions, by rigorously stacking the corresponding time series, provided in SINEX format. Using CATREF software, the datum definition is implemented under minimum condition over a set of high quality EPN sites.

## 1. Introduction

The ETRS89 definition stipulates that its realizations should be co-moving with the rigid part of the Eurasian tectonic plate and should be coincident with the International Terrestrial Reference System (ITRS) at epoch 1989.0, (Boucher and Altamimi, 1992, Altamimi and Boucher 2001). It should be noted that the ETRS89 is linked to the ITRS through rigorous geodetic transformation formula. Therefore the ETRS89 conserves all the global characteristics of the ITRS, because when using the rigorous transformation formula one can go from one to another system without ambiguity. On other words, ITRS and ETRS89 are conceptually identical, differing by conventional transformation formula. One of the major goal of the EUREF Permanent Network (EPN) is to be the backbone of the long-term maintenance of the ETRS89. Therefore precise station positions and velocities as result of weekly EPN solution combination will be the basis of the DEVF.

## 2. Analysis Strategy

CATREF software developed for ITRF activities is used for combination and analysis of the EPN weekly solutions. CATREF contains several modules for handling constraints, comparisons, and combination and analysis of individual terrestrial reference frames provided in SINEX format. Using a least-squares approach, the combination model adjusts simultaneously station positions, velocities, and the sets of Helmert transformation parameters relating each individual solution to the combined frame.

The analysis strategy of the time series combination (rigorously stacking) is based on the following steps:

- original constraints embedded in the weekly SINEX files are removed and replaced by minimum constraints as described below.
- a reference set of EPN stations (plotted in Figure 1) is selected to be used in the datum definition of the combined frame. Station positions and velocities of the reference set are extracted from ITRF2000 solution
- a row combination is first computed in order to identify outliers and jumps for efficient handling of discontinuities
- the final refined combination is first expressed in the ITRF2000 and then converted into the ETRS89, via ITRF2000 parameters as described in Boucher and Altamimi (2001).

The combination model of CATREF software is based on a generalized 7-parameter similarity. Assuming that for each individual solution  $s$ , and each point  $i$ , we have position  $X_s^i$  at epoch  $t_s^i$  and velocity  $\dot{X}_s^i$ , expressed in a given TRF  $k$ , the combination consists in estimating:

- Positions  $X_c^i$  at a given epoch  $t_0$  and velocities  $\dot{X}_c^i$ , expressed in the combined frame  $C$ ;
- Transformation parameters  $T_k$ ,  $D_k$  and  $R_k$  at an epoch  $t_k$  and their rates  $\dot{T}_k$ ,  $\dot{D}_k$  and  $\dot{R}_k$  from the combined frame  $C$  to each individual frame  $k$ .

The general combination model is given by the following equation:

$$\left\{ \begin{array}{l} X_s^i = X_c^i + (t_s^i - t_0)\dot{X}_c^i \\ \quad + T_k + D_k X_c^i + R_k X_c^i \\ \quad + (t_s^i - t_k) [\dot{T}_k + \dot{D}_k X_c^i + \dot{R}_k X_c^i] \\ \dot{X}_s^i = \dot{X}_c^i + \dot{T}_k + \dot{D}_k X_c^i + \dot{R}_k X_c^i \end{array} \right. \quad (1)$$

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where for each individual frame  $k$ ,  $D_k$ ,  $T_k$  and  $R_k$  are respectively the scale factor, the translation vector and the rotation matrix. However, the normal equation constructed using the above set of equations is singular, having a rank deficiency of 14, corresponding to the datum definition parameters. There are several ways implemented in CATREF software to define the combined frame. An efficient way is to use an equation of minimum constraints, given by:

$$(A^T A)^{-1} A^T (X_R - X_c) = 0 \quad (2)$$

where  $X_c$  is the vector of estimated station positions and velocities,  $X_R$  is the reference solution containing a selected set of stations and  $A$  is the design matrix of partial derivatives. Unlike the classical constraints applied over station coordinates, minimum constraints are applied over the frame parameters, thus allowing to express the combined solution in any external frame (e.g. ITRF2000), without altering the quality (or internal consistency) of the estimated solution. For more practical details see for instance (Altamimi, 2003).

### 3. A combination test

As a test combination, we used the EUREF combined weekly solutions of the EPN. Figure 1 shows

the distribution of the used EPN stations. We used the raw combination outputs of the time series to identify outliers and discontinuities. After outlier rejections, discontinuities were handled using break-wise modelling, e.g. considering different station positions, before and after each jump. Velocity before and after the jump is constrained to be the same.

After properly handling the discontinuities and seasonal variations following (Altamimi, 2004), a refined combination was performed and expressed in the ITRF2000. Figure 2 illustrates the horizontal ETRS89 velocities expressed in the ETRF2000, following Boucher and Altamimi (2001). In addition Figure 3 plots the vertical velocities which are obviously expressed in both ITRF2000 and ETRF2000. Figures 2 and 3 clearly show the Post Glacial Rebound phenomena both in the horizontal as well as the vertical components.

To evaluate the quality of the EUREF weekly combined solutions, we computed the WRMS per week based on the output residuals of station positions as results from the time series (stacking) combination. Figure 4 illustrates the WRMS in the horizontal and vertical components indicating an improvement over time reaching the level of 2 mm in the horizontal and 5 mm in the vertical.

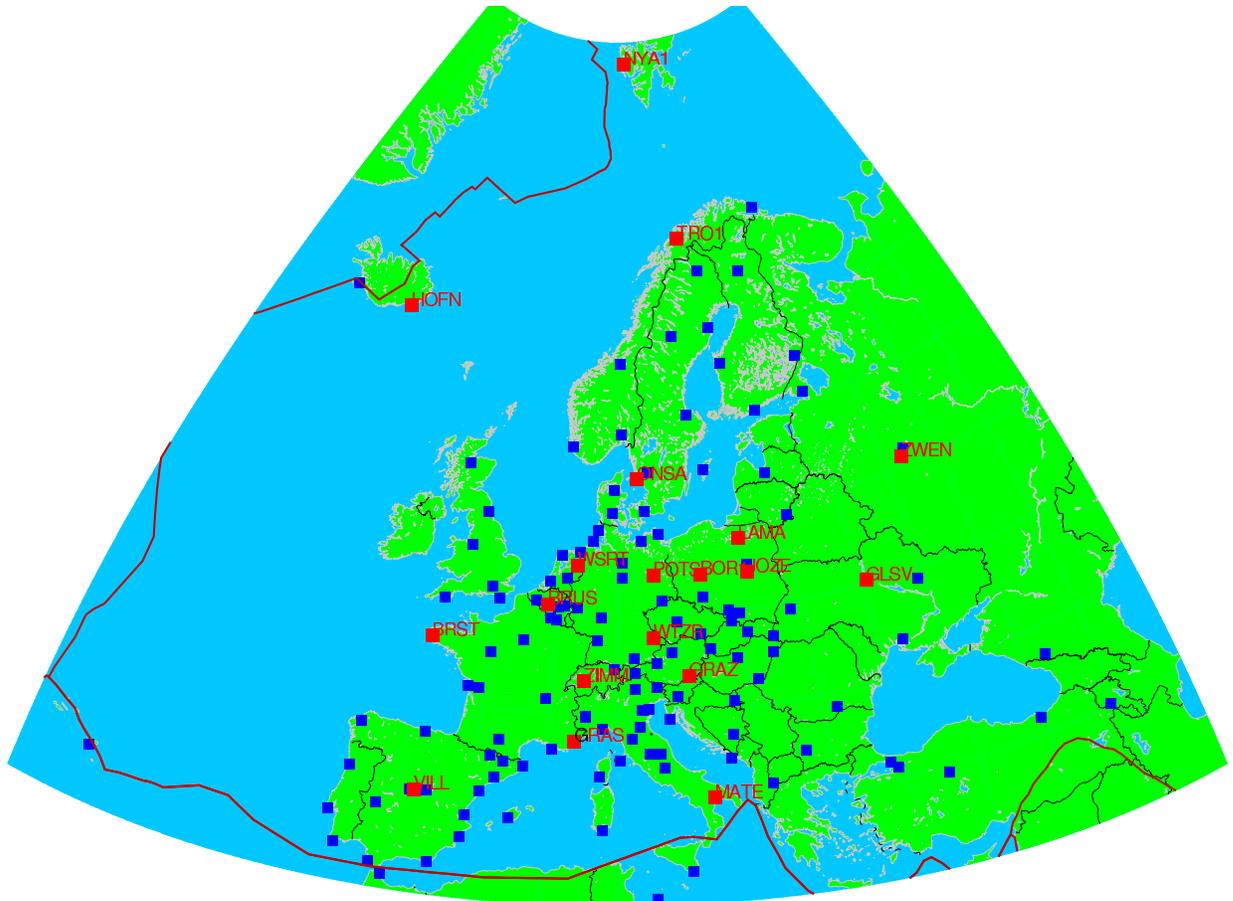


Figure 1. EPN stations underlying the reference stations used in the datum definition.



Figure 2. ETRF2000 horizontal velocities of EPN stations.



Figure 3. ITRF2000/ETRF2000 vertical velocities of EPN stations.

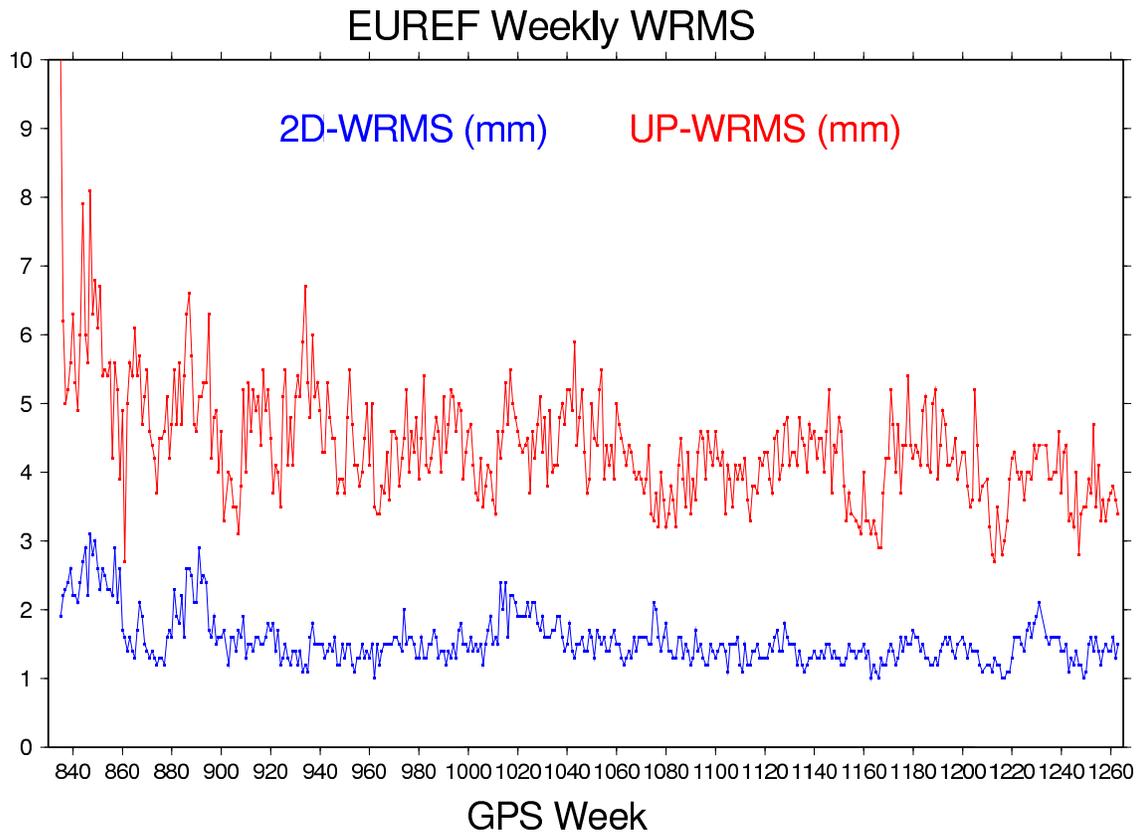


Figure 4. Weekly WRMS of the EUREF combined solutions.

#### 4. Conclusions and future work

The quality of the weekly EUREF combined solution is improving over time and the estimated WRMS is at the level of 2 mm in the horizontal and 5 mm in the vertical components.

In the near future, we intend to make available to users station positions and velocities of the EPN expressed in the ETRS89. This cumulative solution will be updated regularly and will be used as a backbone of the Dense European Velocity Field. Our intention is also to include national and local permanent network weekly solutions as available from the national authorities.

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