Impact of the Combined GPS + Galileo Satellite Geometry on Positioning Precision

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Abstract

The improvement in positioning precision that can be expected from extending the GPS constellation with Galileo satellites will be treated in this paper. The influence of the pure geometric aspect, i.e. the effect of having more available satellites, will be assessed, while the effect of the availability of new signals will be ignored. From this point of view, two scenarios have been studied: (1) the case of stand-alone positioning, typically based on code observables, and (2) the case of relative positioning, typically based on double difference carrier phase observables.

The results show that the use of the additional Galileo constellation improves absolute positioning with about 40% in terms of formal errors when simulating urban conditions. For relative positioning, the concept of Relative Dilution of Precision (RDOP) allowed to demonstrate that using GPS+Galileo only half the observation time is sufficient to get similar precisions as with GPS only.

Introduction

The future European GNSS, Galileo, is designed to be interoperable with GPS. Moreover the combination of Galileo and GPS will provide faster, more reliable and more precise positioning in comparison with present results obtained using GPS only. Positioning at locations with bad visibility will become feasible as twice as much satellites will be visible. The enhanced geometry will also improve the precision of positioning. Moreover, in the future, modernized GPS and Galileo will emit signals on 3 frequencies whereas the current GPS system uses only 2 carrier frequencies. This extra frequency will allow a better correction of the atmospheric disturbances, one of the most important error sources for high accuracy positioning.
The new Galileo system will comprise a constellation of 30 satellites (27 operational and 3 spares) distributed over three circular orbits at an orbital altitude of nearly 24.000 km above the semi-major axis of the WGS84 reference ellipsoid. Galileo uses the same positioning techniques as those used by GPS. But to make fully use of the new GPS and Galileo signals and enhance positioning, algorithms must be adjusted or created to take the new signals into account.

This paper studies the effect of the geometry of the future GPS+Galileo constellation on the precision of the estimated positioning parameters and makes a comparison with the current positioning based on GPS. The computations are design studies only based on the geometry of the GPS and Galileo constellations. The GPS orbits have been created using the broadcast navigation message, while Galileo orbits were simulated for 27 operational satellites using following orbital parameters: a semi-major axis of 29.994 km, an inclination angle of 56°, the eccentricity equal to 0, right ascension angles of -120°, 0° and 120°, a rate of right ascension of 0° a day, the argument of perigee equal to 0°, a mean anomaly with -160°, -120°, -80°, -40°, 0°, 40°, 80°, 120°, 160° as possible values, and finally a period of

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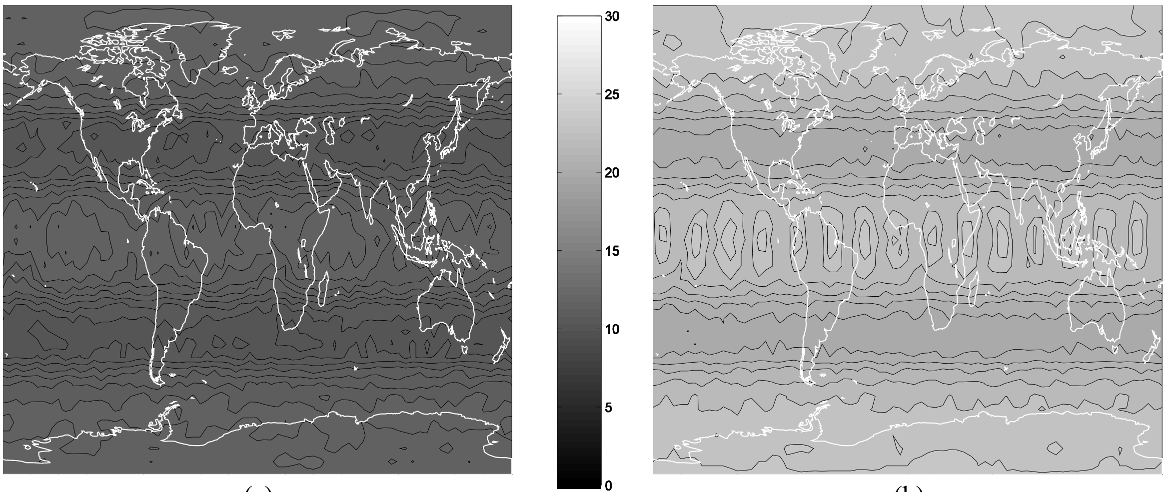
14 h 4 min 42s. We also assume that the GPS and Galileo code observables have the same standard deviation as well as the GPS and Galileo phase observables.

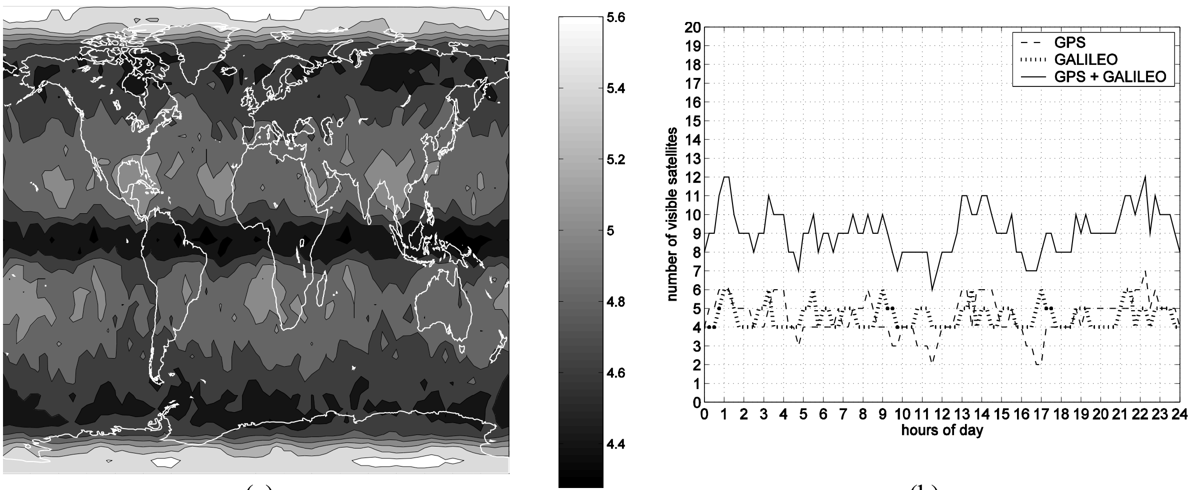
1. Increase of visibility – twice as much satellites

In a first step, the increase of the number of visible satellites from any given receiver position on earth is examined. Figure 1 shows the daily mean of the number of visible satellites for any point on the earth's ellipsoid. Using an elevation cut off angle of 0º, these mean values are computed using a grid of 10º×10º. For the first plot, only the GPS system was considered, while the second plot represents the results for the combined GPS + Galileo systems. The GPS system provides a worldwide mean of about 10 to 12 visible satellites over a day. For the combined system, an increased worldwide distribution of visible satellites, more precisely 21 to 23, is seen.



(a) (b)

Figure 1 : Worldwide distribution of the daily mean of visible satellites using a 0° cut off,
(a) GPS only
(b) combined GPS+GALILEO



(a) (b)

Figure 2 : using a 30° cut off,
(a) Worldwide distribution of the daily mean of visible GPS satellites
(b) Daily number of visible satellites in Reykjavik for all systems

Adding Galileo to GPS will make especially a difference when we have a blocked horizon (e.g. in cities). In this case, a cut off of 30º is representative. Figure 2a emphasizes locations where point-positioning is not possible due to a lack of visible satellites at certain epochs using GPS only. An example of such a problem location is shown in Figure 2b, which shows a plot of the number of satellites visible over a day in the EPN station at Reykjavik, Iceland. We can see that there are epochs where only 2 or 3 satellites are visible, making it impossible to calculate a position. With Galileo only, a minimum of at least 4 satellites is always visible (even in our conservative case where we did not consider the 3 spare satellites), whereas the combined system always provides more than enough satellites.

2. Stand-alone positioning based on code observables

Stand-alone positioning using GPS code observables is utterly useful for navigation applications with a precision of several meters. At any time**,** the code observable R_p^i is obtained by measuring the transmission time τ_p^i of the code emitted by satellite i and received by receiver p . This transmission time is the difference between time of arrival (measured on the receiver clock) and time of emission (measured on the satellite clock) of the signal, and can also be characterized as follows:

$$\tau_p^i = \frac{\rho_p^i}{c} + T_p^i + I_p^i + MP_p^i \tag{1}$$

with ρ_p^i the approximate geometric distance between satellite i and receiver p , c the speed of light, T_p^i and I_p^i the respective tropospherical and ionospherical delays and finally MP_p^i representing the multipath errors and the delay of the electromagnetic signal while passing through the receiver hardware. The pseudo-range measurement at time t can be represented by the following observation equation:

$$R_p^i(t) = \rho_p^i(t) + c(-\delta_p(t) + \delta^i(t)) + I_p^i(t) + T_p^i(t) + MP_p^i(t) + \varepsilon_{R_p} \tag{2}$$

with $\delta_p(t)$ en $\delta^i(t)$ the respective receiver and satellite clock errors and ε_{R_p} the measurement noise.

This study considers an 'ideal' positioning environment, free of the typical errors affecting positioning. This means that we neglect the ionospheric and tropospheric errors I_p^i and T_p^i , as well as the multipath errors. We assume the satellite clock error $\delta^i(t)$ to be known and equal to 0, and consider the simulated Galileo orbits and the orbits given by the GPS navigation message as correct.

This simplified pseudo-range observation equation can not be solved immediately for the position coordinates, but has to be linearized first. Having an a priori receiver position (X_{op}, Y_{op}, Z_{op}), the unknown receiver position (X_p, Y_p, Z_p) can be written as $X_p = X_{0p} + \Delta X_p$, idem for Y_p and Z_p. Next, linearization by a Taylor series expansion will be made around $(\Delta X_p, \Delta Y_p, \Delta Z_p)$, those parameters becoming our new unknowns. Consequently the pseudo-range observation model becomes:

$$R_p^i(t) - \rho_{op}^i(t) = -\frac{X^i(t) - X_{op}(t)}{\rho_{op}^i(t)}\Delta X_p - \frac{Y^i(t) - Y_{op}(t)}{\rho_{op}^i(t)}\Delta Y_p - \frac{Z^i(t) - Z_{op}(t)}{\rho_{op}^i(t)}\Delta Z_p - c\delta_p(t) + \varepsilon_{R_p} \tag{3}$$

$\rho_{op}^i(t)$ is the approximate geometric distance between satellite i and receiver p calculated with the a priori receiver position. Writing Equation (3) in matrix notations gives $L = AX + v$, with X the vector $(\Delta X_p, \Delta Y_p, \Delta Z_p, \delta_p)$ of the unknowns. Each element of the observations vector L equals $R_p^i(t) - \rho_{op}^i(t)$ and the design matrix A consists of the coefficients of the unknowns in each observation equation. Finally, v is the vector of the residuals.

Given the positions of the visible satellites, the observation equation system can be solved using the Least Squares Method. Minimizing the weighted sum of the residuals $(v^T P v)$, results in the Least Squares Solution $X = (A^T PA)^{-1} A^T PL$. The weight matrix P is defined as $P = \sigma_0^{-2} \Sigma_L^{-1}$ with σ_0^2 the a priori variance and Σ_L^{-1} the inverse of the covariance matrix of the observations. For code-based positioning, we will consider a unit matrix as the covariance matrix of the observations Σ_L. The covariance matrix of the unknowns is $\Sigma_X = (A^T \Sigma_L^{-1} A)^{-1} = (A^T A)^{-1}$, and its diagonal elements provide us information on the formal errors of the estimated parameters.
After transforming the matrix Σ_X into its topocentric equivalent covariance matrix, using the law of variance propagation, we obtain

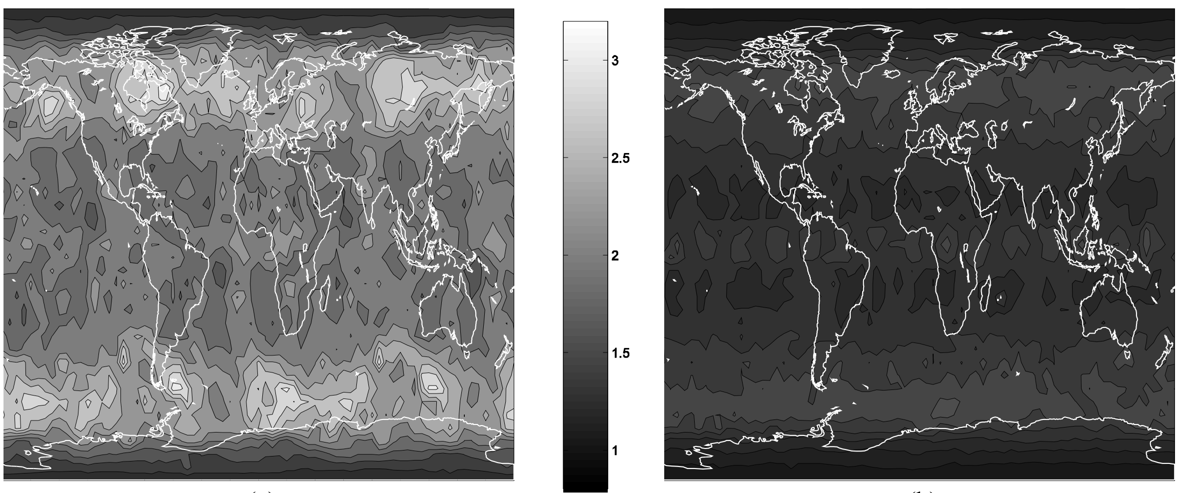
$$\Sigma_T = R\Sigma_X R^T = \begin{bmatrix} \sigma_n^2 & \sigma_{ne}^2 & \sigma_{nu}^2 \\ \sigma_{en}^2 & \sigma_e^2 & \sigma_{eu}^2 \\ \sigma_{un}^2 & \sigma_{ue}^2 & \sigma_u^2 \end{bmatrix} \tag{4}$$

with R the rotation matrix.
This covariance matrix depends only on the geometry of the visible satellites and allows to extract Dilution of Precision (DOP) values (*Husti, 2000*):

$$\begin{cases} HDOP = \sqrt{\sigma_n^2 + \sigma_e^2} \\ VDOP = \sigma_u \\ PDOP = \sqrt{\sigma_n^2 + \sigma_e^2 + \sigma_u^2} \end{cases} \tag{5}$$

referring respectively to the horizontal component, the vertical component and the 3-dimensional positioning.

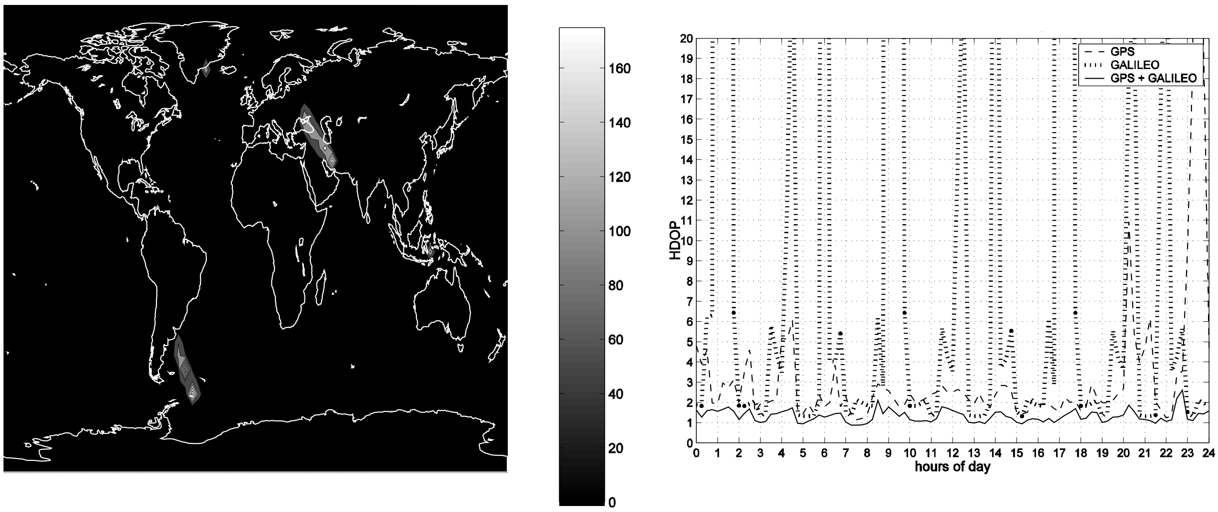
Figure 3 shows the worldwide $HDOP$ values in simulated urban conditions with a cut off angle of 30°.



(a) (b)

Figure 3 : Worldwide distribution of the daily median of *HDOP* value using a 30° cut off
(a) GPS only
(b) combined GPS+GALILEO

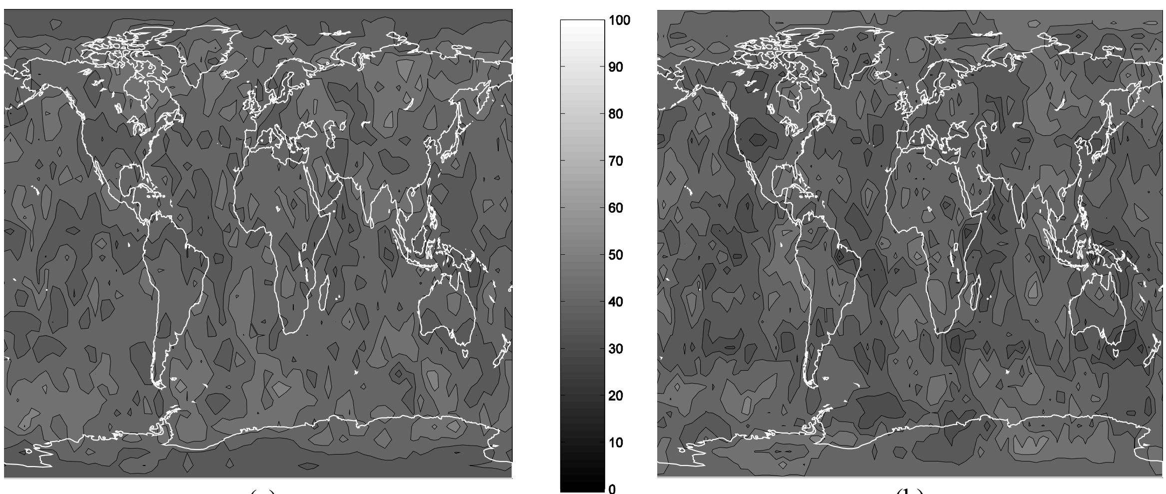
To eliminate the influence of $HDOP$ outliers, median values are shown instead of means. The results for the combined system show $HDOP$ values below 1.5. For GPS only, most of the worldwide values are less than 2.5. That seems to be a good value, but for some locations problems occur when considering the $HDOP$ value over the whole day. As an example, the daily evolution of the $HDOP$ value for the EPN station Zelenchukskaya in Russia, is showed in Figure 4b. This station was chosen because in Figure 3a as well as in Figure 4a, it lies respectively in an area with high –nevertheless acceptable– median $HDOP$ values and in an area with huge mean $HDOP$ values.



(a) (b)

Figure 4 : using a 30° cut off
(a) Worldwide distribution of the daily mean of the *HDOP* value with GPS
(b) Daily distribution of *HDOP* value in Zelenchukskaya for all systems

In Zelenchukskaya, very huge daily mean $HDOP$ values of 51.47 and 721.66 have been observed for both GPS only and Galileo only. These very bad values are caused by a lack of visible satellites at certain epochs, explaining the outliers $(HDOP > 20)$ also visible in Figure 4b. Respectively 1 and 15 outliers for GPS only and Galileo only are visible in Figure 4b. On the other hand for the hybrid system, no outliers and not even $HDOP$ values over 3 were observed, yielding a very good daily mean $HDOP$ value of 1.33.



(a) (b)

Figure 5 : using a 30° cut off
(a) Worldwide distribution of improvement ratio for daily median of the *HDOP* value
(b) Worldwide distribution of improvement ratio for daily median of the *VDOP* value

Going back to the results showed in Figure 3, we conclude that the worldwide decrease of $HDOP$ from less than 2.5 to less than 1.5, corresponds with an improvement of about 40% on the formal errors of the horizontal components of the station position when using a combined GPS+Galileo constellation compared to GPS only (see Figure 5a). Figure 5b shows the worldwide improvement for the $VDOP$ which is similar to the $HDOP$ results.

3. Relative positioning, based on double difference carrier phase observables

Relative positioning differs from absolute positioning because the vector $(X_{pq}, Y_{pq}, Z_{pq}) = (X_p - X_q, Y_p - Y_q, Z_p - Z_q)$ between two receivers p and q is calculated instead of one single receiver position. Relative positioning requires the introduction of single and double differences $(SD \,\&\, DD)$. These differences decrease or eliminate the influence of some of the error sources. The DD carrier phase observation equation between receivers p and q and satellites i and j is:

$$\Phi^{ij}_{pq}(t) = \Phi^{i}_{pq}(t) - \Phi^{j}_{pq}(t) = \rho^{ij}_{pq}(t) + \lambda N^{ij}_{pq} + I^{ij}_{pq} + T^{ij}_{pq} + \varepsilon_{\Phi^{ij}_{pq}} \tag{6}$$

with Φ^{ij}_{pq} the DD of the carrier phase observable, ρ^{ij}_{pq} the DD of the approximate geometric distances between satellites i and j and receivers p and q, λ the wavelength of the signal, while N^{ij}_{pq} is the DD of the integer initial ambiguities. The first order differential ionospheric effect I^{ij}_{pq} is eliminated because we assume the use of an ionosphere free combination. On the other hand the tropospherical delay T^{ij}_{pq} is considered known. All other error sources, e.g. multipath, are part of the measurement noise.

Similar to absolute positioning, the carrier phase observation Equation (6) is linearized in order to solve for the unknown parameters. Once more, the receiver position will be written as $(X_R, Y_R, Z_R) = (X_{0R} + \Delta X_R, Y_{0R} + \Delta Y_R, Z_{0R} + \Delta Z_R)$ given an a priori estimated position (X_{oR}, Y_{oR}, Z_{oR}). Further, Taylor series expansion will be executed around $(\Delta X_p, \Delta Y_p, \Delta Z_p)$ and $(\Delta X_q, \Delta Y_q, \Delta Z_q)$ for respective terms belonging to receivers p and q. Taking receiver p as the reference station, its coordinates are known and consequently $\Delta X_p = \Delta Y_p = \Delta Z_p = 0$. In this test, we consider the initial ambiguities N^{ij}_{pq} as being fixed, yielding to following observation model:

$$\Phi^{ij}_{pq}(t) - \rho^{ij}_{opq}(t) - \lambda N^{ij}_{pq} = a^{ij}_{Xq}(t)\Delta X_q + a^{ij}_{Yq}(t)\Delta Y_q + a^{ij}_{Zq}(t)\Delta Z_q + v \tag{7}$$

The coefficients $a^{ij}_{Xq}(t)$, $a^{ij}_{Yq}(t)$ and $a^{ij}_{Zq}(t)$ of the unknowns, are then given as:

$$\begin{cases} a^{ij}_{Xq}(t) = \left[\dfrac{X^i(t) - X_{oq}}{\rho^i_{oq}(t)} - \dfrac{X^j(t) - X_{oq}}{\rho^j_{oq}(t)} \right] \\ a^{ij}_{Yq}(t) = \left[\dfrac{Y^i(t) - Y_{oq}}{\rho^i_{oq}(t)} - \dfrac{Y^j(t) - Y_{oq}}{\rho^j_{oq}(t)} \right] \\ a^{ij}_{Zq}(t) = \left[\dfrac{Z^i(t) - Z_{oq}}{\rho^i_{oq}(t)} - \dfrac{Z^j(t) - Z_{oq}}{\rho^i_{oq}(t)} \right] \end{cases} \tag{8}$$

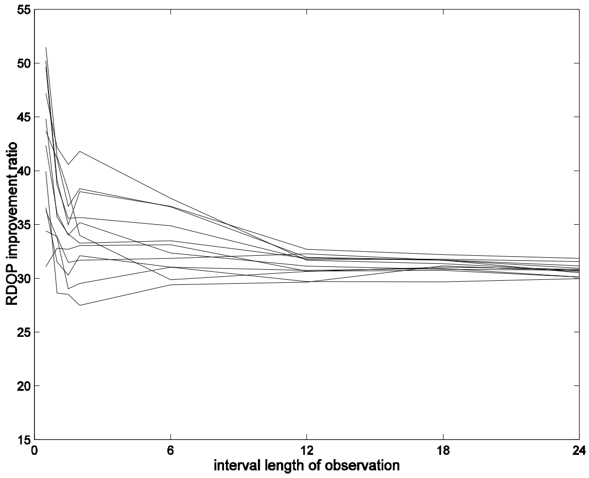
Every observation can be written as Equation (7), yielding a model represented by the matrix equation $L = AX + v$. X is the vector $(\Delta X_q, \Delta Y_q, \Delta Z_q)$ of the unknowns, A the design matrix containing all coefficients of the unknowns, L the vector containing all observations and finally v is the vector of residuals.

Using the observation model from Equation (7), we can compute the associated covariance matrix of the unknowns $\Sigma_X = (A^T \Sigma_L^{-1} A)^{-1}$ and convert it to its topocentric equivalent Σ_T, similar to what was done for absolute positioning. The covariance matrix of the observables Σ_L is not a unit matrix, but the mathematical correlations between the double difference measurements are now taken into account.
Again, the covariance matrix of the unknowns provides information on the precision of the solution. The $RDOP$ (Relative DOP) is similar to the $PDOP$ value for the case of absolute positioning, but will be calculated in a different way with the formula (*Goad, 1988*):

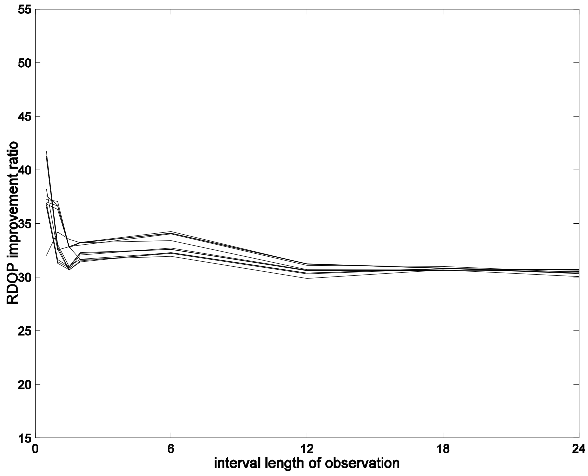
$$RDOP = \sqrt{\frac{trace(\Sigma_X)}{\sigma_{DD}^2}} \tag{9}$$

In addition, contrary to the computation of the $PDOP$, the observations will now be accumulated over sessions varying between ½ and 24 hours, using a 60 seconds measurement interval. Going back to formula (9), σ_{DD} is the uncertainty of a DD measurement. This definition implies that the $RDOP$ will not depend on the a priori variance σ^2 of the carrier phase measurements. For the matrix Σ_X as well as for the value σ_{DD}^2, a factor σ^2 can be set apart, whereas those factors appearing in denominator and nominator of Equation (9) can be removed. Consequently, we will not have to make assumptions about this value. The units of $RDOP$ are meters/cycle. Theoretically, the uncertainty of a DD measurement multiplied by $RDOP$ will therefore yield a relative position error.

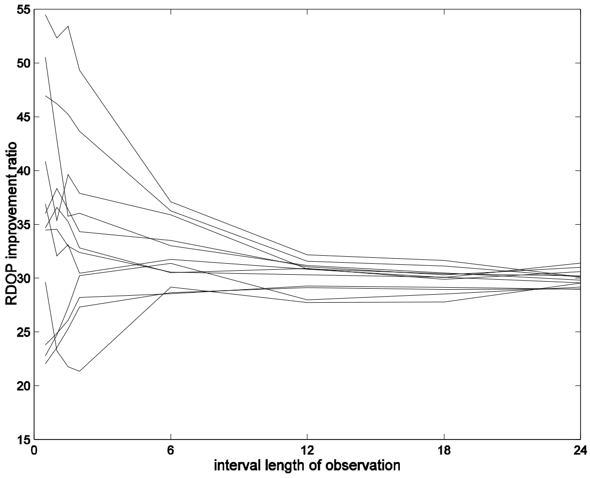
Following the same principle as for $HDOP$ and $VDOP$, we considered north, east and up components for $RDOP$. We have chosen two sets of baselines between EPN stations, long distances of about 1000-4500km (Figure 6a) and short distances of about 40-350km (Figure 6b), each set of observations divided in three kinds of pairs: stations with equal latitude, stations with equal longitude and randomly chosen pairs. The observations accumulated over sessions of 12 hours show the same improvement for north, east as well as for up components. This improvement is about 30% for both long and short baselines in comparison with previous results obtained for GPS only.



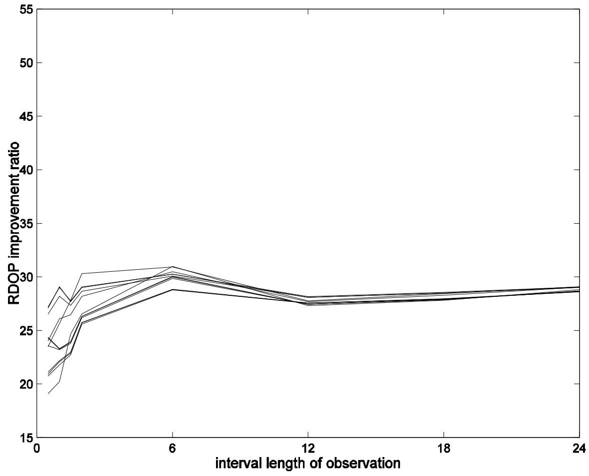
(a) North component



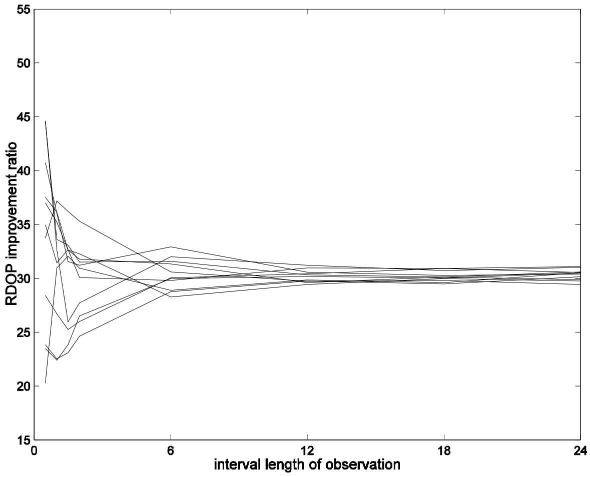
(b) North component



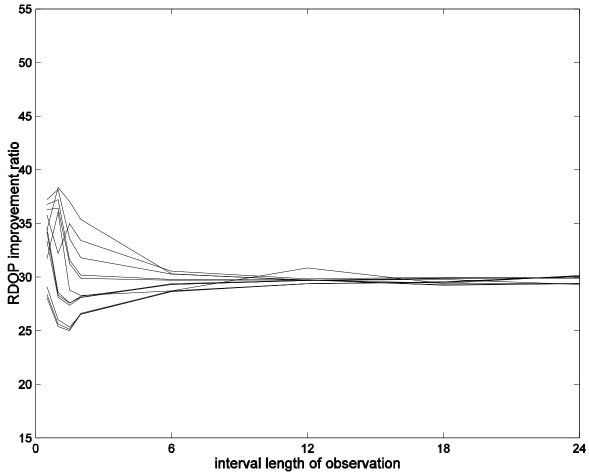
(a) East component



(b) East component



(a) Up component



(b) Up component

Figure 6 : RDOP improvement ratio
(a) For baselines of about 1000-4500 km
(b) For baselines of about 40-350 km

Another interesting result is that the same RDOP values can be obtained for the hybrid system in only half of the observation time needed when using GPS only. Table 1 illustrates this result for the example EPN-stations Brussels and La Palma.

tijdsinterval	RDOP_gps	RDOP_gg
1/2 u	0.2439	0.1733
1 u	0.1788	0.1188
3/2 u	0.1326	0.0918
2 u	0.1141	0.0794
6 u	0.0617	0.0438
12 u	0.0425	0.0303
18 u	0.0351	0.0248
24 u	0.0307	0.0215

Table 1 : RDOP values for GPS only and for the hybrid system for varying interval lengths

Conclusion

This paper compared the formal errors of absolute and relative positioning performed using a GPS only satellite constellation and using a combined GPS+Galileo constellation. As expected, using GPS+Galileo more visible satellites are available worldwide and the combined system reduces or even eliminates the presence of problem locations worldwide.
Using the DOP values, we demonstrated that the contribution of the satellite geometry to the total positioning error budget is reduced with about 40% only by adding the Galileo constellation to the GPS constellations. For relative positioning, we showed that the combined system reaches similar formal errors as stand-alone GPS, but using only half of the observation time in comparison with GPS only**.** As mentioned before, these results are only valid in an ideal environment because the effect of other error sources on the total error budget has been neglected.

References

C.C. Goad, Investigation Of An Alternate Method Of Processing Global Positioning Survey Data Collected In Kinematic Mode, in *GPS-Techniques Applied to Geodesy and Surveying*, April 1988.

G.J. Husti, *Global Positioning System, een inleiding*, 2000.